Multi-Cellular Natural Convection in the Melt during Convection-Dominated Melting

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Convection-dominated melting in a rectangular cavity is analyzed numerically with particular attention to the multi-cellular flows in the melt. At the earlier stage of the melting, the melt region is quite similar to a cavity with high aspect ratio, where the multi-cellular natural convection appears. Numerical results show that the formation and evolution of the multiple flow cells in the melt region is approximately similar to that of a single-phase flow in a tall cavity with the same aspect ratio; however, the continuous change of the melt region due to the melting affects the detailed process. Also, numerical aspects for the prediction of the detailed flow structure in the melt are discussed.

Key Words : Melting, Natural Convection, Multi-Cellular Pattern, Enthalpy-Porosity Method

Nomenclature

A : Aspect ratio, $A=H/W$ [-]
b : Constant for Darcy-like momentum source term [-]
C : Constant for Darcy-like momentum source term [kg/m^3 s]
c : Specific heat [J/kg K]
f : Liquid fraction [-]
Gr_{crit} : Critical Grashoff number at which multi-cellular pattern emerges [-]
Gr_W : Grashoff number based on cavity width, $Gr_W=\frac{g\beta(T_H-T_m)W^3}{\nu^2}$ [-]
H : Cavity height [m]
P : Pressure [Pa]
Pr : Prandtl number, $Pr=\frac{\nu}{\alpha}$ [-]
Ra : Rayleigh number, $Ra=\frac{g\beta(T_H-T_m)H^3}{\alpha\nu}$ [-]
$\vec{S}$: Momentum source vector in enthalpy-porosity model [m/s^2]
T : Temperature [K]
T_C : Cold wall temperature (right wall) [K]
T_H : Hot wall temperature (left wall) [K]
T_i : Initial temperature [K]
T_m : Melting temperature [K]
$\vec{u}$: Velocity vector [m/s]
W : Cavity width [m]

Greek symbol

α : Thermal diffusivity [m^2/s]
β : Volumetric compressibility referred to the reference temperature [1/K]
δ_{crit} : Critical thickness at which multi-cellular flow pattern apprears [m]
ν : Kinematic viscosity [m^2/s]
ρ : Density [kg/ m^3]
ψ : Stream function [m^2/s]

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1. Introduction

In many engineering fields such as thermal energy storage systems using latent heat and materials processing, solid-liquid phase-change is an important process for the optimal design and operation. The phase change is strongly related to

the temperature distribution in the liquid phase as well as in the solid phase and even small temperature differences in the melt can give rise to the natural convection due to buoyancy forces. The convection gives a great influence on the morphology of the solid-liquid interface, which could significantly affect the flow structure in the melt (Voller, 1997).

Although there have been considerable efforts to the analysis of the convection-dominated melting, relatively little research has been conducted on the minor structure of the melt convection. Dantzig (1989) first discussed the detailed structure in the convection-dominated melt. He conducted numerical simulations of the solid-liquid phase change with melt convection using FIDAP, which is a well-known commercial code based on the finite element method. Using temporal enthalpy-specific heat formulation with the enhanced viscosity model, Dantzig observed that multiple flow cells were formed in the melt at the earlier stage of the melting process and they merged as the melting continued. After the simulation with the streamline upwinding scheme for the convection terms in the momentum and energy transport equations, he concluded that the numerical modeling of the convection term is important to capture the minor structure in the melt. Recently, Rady et al. (1997) observed a multiple flow pattern when they carried out a numerical simulation for the solidification process. Two circulation cells were captured in the thin melt region at the later stage of solidification, where the aspect ratio of melt zone was about seven. Stella and Giangi (2000) and Wintruff et al. (2001) contributed to numerical investigations on the multi-cell formation at the earlier stage of the melting. While Stella and Giangi obtanined six flow cells as Danzig did, Wintruff et al. observed four cells. Although their results are somewhat different in the evolution pattern of melt flow, there is consistence that a multi-cellular flow structure is built up during the initial stage of melting and evolves, then eventually decays.

As for single-phase flow, since the multi-cellular flow structure in a tall cavity was first observed and reported by Elder (1965), there have been many attempts to determine the dependence of the flow structure on the aspect ratio and on the Rayleigh number. Lee and Korpela (1983) conducted a systematic numerical investigation for multi-cellular flow patterns and the effects on the heat transfer in various tall vertical enclosures over a wide spectrum of Prandtl numbers (from 0 to 1000). They found the critical value of the Grashof number at the moment the secondary cells occur. Wakitani (1998) was interested in the number of cells in the center region according to the Rayleigh number. He conducted numerical simulations for a vertical air-filled tall cavity to clarify the role of Rayleigh number and aspect ratio on the flow structure. He observed the transition from the onset of the steady multi-cellular flow, through the reverse transition to the unicellular pattern, to the unsteady multi-cellular flow. Recently, Lartigue et al. (2000) conducted a numerical and experimental study to characterize the temperature field, the associated Nusselt number, and the velocity field in a high aspect ratio cavity. They focused on the evolution of the multi-cellular patterns in the core of the cavity and found that cells do not remain stationary but move downward for a certain aspect ratio and Rayleigh number.

The main objective of this paper is to present the numerical results for the detailed flow structure in the molten phase during natural convection-dominated melting in a rectangular cavity. In order to produce these results, the single-domain approach is employed to describe the momentum and energy fields over both solid-phase and liquid-phase regions with a single set of governing equations and boundary conditions. A pertinent source term that will be determined by nodal enthalpy is added to the energy equation to account for the latent heat absorption and evolution. The Navier-Stokes equation is modified to suppress the motion in the solid phase by adding a source term, which is similar to the Darcy term for a a porous medium analysis. The enthalpy-based single-domain method combined with the porous medium approach is

often referred as the enthalpy-porosity method. The spatial and temporal discretizations are based on the finite volume scheme and the fully implicit (backward) Euler scheme, respectively. The flow field is expressed in terms of primitive variables, which are obtained by adopting SIMPLE algorithm (Ferziger and Peric, 1999). In regard with the interpolation of the advection term, which is known to be essential in the calculation of the flow structure in a tall cavity, we use the deferred correction method (Khosla and Rubin, 1974) based on the upwind and central difference schemes as a lower and a higher order interpolation, respectively. The influence of the interpolation scheme on the numerical predictability of the detailed flow structure is also discussed.

2. Mathematical and Numerical Models

To illustrate two-dimensional melting driven by the natural convection and conduction, a rectangular cavity that is schematically described in Fig. 1 is considered. The phase change material is contained in a cavity, whose vertical sides are maintained at constant temperatures, while the connecting horizontal walls are adiabatic. Initially, the phase change material in the cavity is kept at uniform temperature below or at the fusion temperature, $T_i \leq T_m$, where T_i is the initial temperature and T_m is the melting temperature. The melting is initiated by raising the temperature of the left wall abruptly to a predetermined temperature, T_H, above the fusion temperature, $T_H > T_m$. The temperature at the right wall, T_C, is maintained at the initial temperature, $T_C = T_i$. As time elapses, the buoyancy force becomes large enough to overcome viscous retardation and triggers natural convection in the melt zone.

The momentum field is subjected to no-slip boundary conditions at the walls. The flow is assumed to be two-dimensional, laminar, and incompressible. The thermophysical properties of materials are constant, but may be different for liquid and solid phases. The density difference

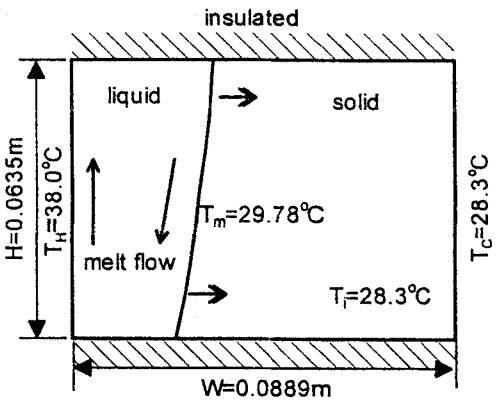


Fig. 1 Description of gallium melting experimented by Gau and Viskanta(1986) ($Ra=6.057\times10^5$, $Pr=0.0216$)

between solid and liquid phases is neglected except when invoking the Boussinesq approximation. The phase-change process is simulated by adopting the enthalpy-porosity model, where the absorption and evolution of latent heat during phase change is modeled as a source term in the energy equation. Also, a Darcy-like term (Viswanath and Jaluria, 1993) is introduced to suppress the velocity at the computational cell located in the solid phase should be suppressed while the velocities in the liquid phase remains unaffected. Then, the governing equations to solve the melting phase change will be

$$\nabla\cdot\vec{u}=0, \tag{1}$$

$$\frac{\partial\vec{u}}{\partial t}+\vec{u}\cdot\nabla\vec{u}=-\frac{1}{\rho}\nabla p+\nu\nabla^2\vec{u}+\vec{g}\beta(T-T_m)+\vec{S}, \tag{2}$$

$$\frac{\partial T}{\partial t}+\vec{u}\cdot\nabla T=\alpha\nabla^2 T-\frac{L}{c}\frac{\partial f}{\partial t} \tag{3}$$

In this, $\vec{S}$ is a Darcy-like momentum source term defined as

$$\vec{S}=-C\frac{(1-f)^2}{(f^3+b)}\vec{u} \tag{4}$$

where C is a big constant to suppress the velocity as a cell becomes solid and b is a small number used to prevent the division-by-zero when a cell is fully located in the solid region, namely $f=0$. In this work, $C=1\times10^9\text{kg/m}^3\text{s}$ and $b=0.005$ are used (Viswanath and Jaluria, 1993).

For the effective calculation, the present study employs the front-layer predictor-corrector and

the pseudo Newton-Raphson algorithms, which were devised for the phase-change heat conduction problem (Kim et al., 2001a) and successfully applied to the convection-dominated phase-change problem (Kim et al., 2001b). The detailed description of the mathematical model can be found in Kim et al. (2001b) and is omitted for the brevity.

The SIMPLE algorithm (Ferziger and Peric, 1999) is used to find the velocity and pressure field. For the interpolation of convection term, this study adopts the central difference scheme.

3. Numerical Results and Discussions

3.1 Natural convection in a tall cavity

Before solving the phase change problems, we consider a single-phase natural ? convection problem in a tall cavity. Through the preliminarty calculation, we will grasp the numerical issues relating to the prediction of the multi-cellular flow patterns, as well as verify the predictability of the numerical model developed previously (Kim et al., 2001b).

Lee and Korpela (1983) gave an extensive discussion on the multi-cellular nature of the natural convection in a cavity with high aspect ratio. They presented the computational results of the natural convection in a tall cavity ($A=15$) for $Pr=0$ and, especially, observed a transition from a unicellular to a multi-cellular flow at $Gr_W \approx 8000$, where Gr_W denotes the Grashof number based on the width of the cavity. Even though their results for the zero-Prandtl number fluid seem to be ideal, it is known that the Prandlt number within the range of liquid metals little affects the nature of the natural convection (Lee and Korpela, 1983).

The schematic of the problem is summarized in Fig. 2(a). The left (hot) and right (cold) walls are kept at constant temperatures while the top and bottom sides are insulated. The initial temperature is set to the cold wall temperature. Figure 2(b)-(d) show the streamline contours obtained in a vertical cavity with an aspect ratio of 15 for $Gr_W=5000$, 8000 and 10000. The Prandtl number is set to $Pr=0.01$. The streamlines obtained with the upwind difference scheme are given in Fig. 2(e). In order to achieve grid-independent solution and to resolve the multi-cellular structure, several grid systems are tested. Results of a grid refinement study presented in Table 1 show that 25×100 uniform grid gives fairly good predictions. As depicted in Fig. 2, 25×100 uniform grid is also sufficient to resolve the multi-cellular structure. The multi-cellular pattern emerges around $Gr_W=8000$ and at $Gr_W=10000$ the multi-cellular structure with six cells is fully established. This observation is exactly identical to that obtained by Lee and Korpela (1983). On the other hand, Fig. 2(e) shows that the upwind difference scheme results in the flow structure with only a single cell. The upwind difference scheme, which is a first order approximation, causes the false diffusion, which hinders the formation of the secondary cells. Thus, the predictability of the code is proven and the diffusive characteristic of upwind difference scheme is reconfirmed.

Table 1 Grid refinement results for natural convection in a cavity of $A=15$ for $Pr=0.01$ and $Gr_W=10000$

Grid (uniform)	20×80	25×100	25×150	35×175
$-\psi_{min}/\alpha$	0.31104	0.30609	0.30656	0.30658

3.2 Multiple flow cells in the melt

Consider the earlier stage of the melting of a pure substance in a rectangular cavity heated from a sidewall. At the beginning of the melting, the heat transfer is governed by the conduction. As the melting proceeds, the temperature difference between the hot wall and the liquid-solid interface will provide the melt with enough force to flow and eventually the natural convection will be initiated. At the earlier stage of the melting, the phase-change front marches nearly parallel to the wall. As long as melt region remains thin, the melt convection may be regarded as a natural convection in a tall cavity. Multiple cells can be formed in this tall cavity. The liquid is confined between the hot wall and the solid-liquid interface. If the constant temperature is imposed

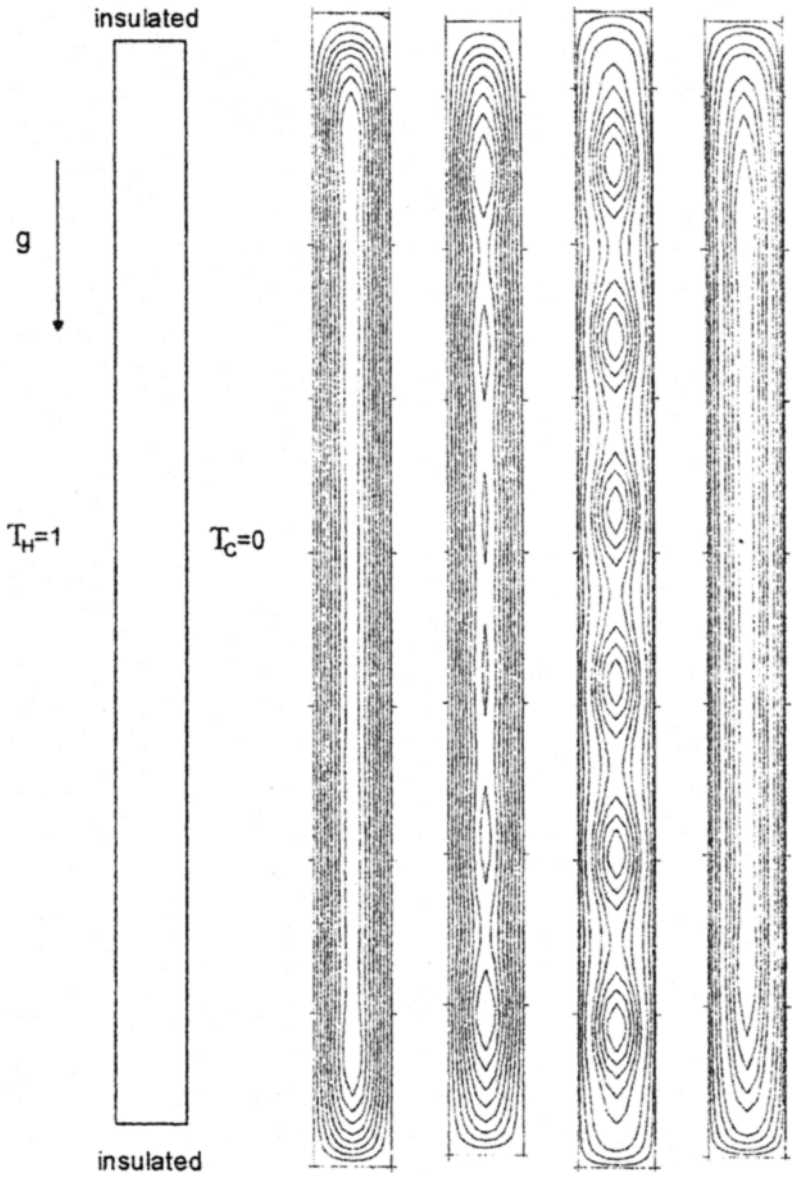


Fig. 2 Natural convection of a fluid with $Pr=0.01$ in a rectangular cavity of aspect ratio $A=15$. Problem description (a) and streamlines for $Gr_w=5000$ (b), $Gr_w=8000$ (c) and $Gr_w=10000$ with central difference scheme (d) and for $Gr_w=10000$ with upwind difference scheme (e)

on the hot wall, the flow structure may be quite similar to the typical natural convection in a cavity with constant sidewall temperatures. This is due to the fact that the temperature at the phase change interface is always kept at the melting temperature.

As obtained by Lee and Korpela (1983) and also in the present study, the multi-cell structure comes out at Grashof number of the order of 8000 ($Gr_w \approx 8000$) in a rectangular cavity with $A=15$. In a melting problem where the thickness of the melt is not definite but varies as the melting progresses, the critical thickness of the melt, δ_{crit}, where the multi-cellular flow cells appear, can be estimated as follows:

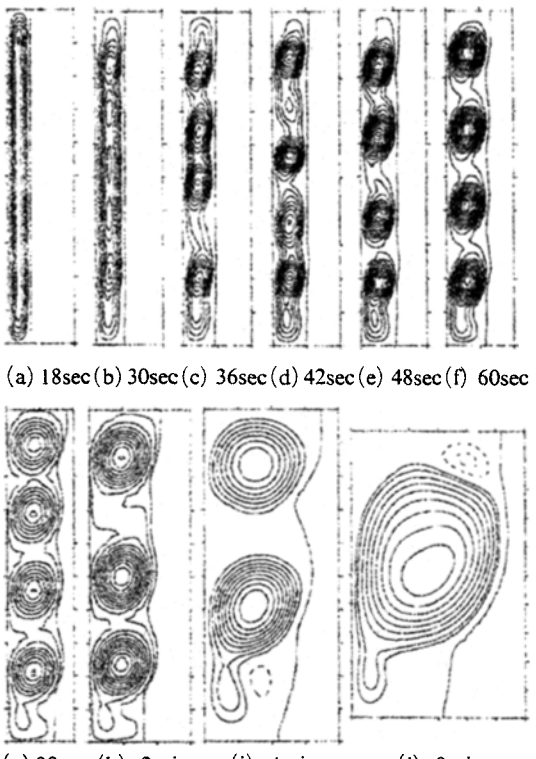
$$Gr_{crit}=\frac{Ra}{Pr}\left(\frac{\delta_{crit}}{H}\right)^3 \quad (5)$$

where Gr_{crit} is the Grashoff number based on the critical thickness. The Rayleigh number Ra is based on the cavity height H. Gau and Viskanta' s data of gallium melting experiment (Gau and Viskanta, 1986), which has been frequently adopted for the model validation and was also simulated by the authors (Kim et al., 2001b), give $H/\delta_{crit}\approx 15$ if $Gr_{crit}\approx 8000$. When the melt thickness reaches the critical value, contingently, the aspect ratio of the melt region corresponds to the tall cavity considered previously for the calculation of the multi-cellular natural convection. Hence, we will investigate the detailed flow structure in the melt under the experimental conditions of Gau and Viskanta's experiments, which are depicted in Fig. 1.

The calculation of multi-cellular natural convection in a tall cavity shows that a sufficient number of grids are required to resolve the multi-cellular pattern. For example, Lee and Korpela (1983) recommended 10 nodes per flow cell in the vertical direction and more than 17 nodes in the horizontal direction. The grid refinement test performed in this study (Table 1) demonstrates that grid systems finer than 25×100 are required for the natural convection problem with $A=15$, $Pr=0.01$, and $Gr_W=10000$. The problem with higher aspect ratio, the Prandtl number and the Grashof number can be resolved only with finer grids.

The grid system is determined on the basis of the above preliminary calculation for a tall cavity as well as the simulations conducted previously by Kim et al. (2001b). They successfully reproduced the movement of the phase front with uniformly spaced 50 ? 36 grid. The vertical direction is divided into 100 uniform grids. Three subdomains are allocated along the horizontal direction: $0\le x/H\le 0.075$, $0.075<x/H\le 0.4$, and $0.4<x/H\le 1.4$. The first subdomain that fairly contains the critical thickness is uniformly divided into 30 computational cells with the size of $\Delta x/H=0.0025$, which is used in the above calculation on the natural convection in a tall cavity. For the second and the third subdomains, 40 and 50 non-uniform (gradually increasing) nodes are allocated, respectively. In doing this, the right-most grid (0.021) is smaller than the ones (0.028) used in the gallium melting calculation, which was proven to give good results on the evolution of phase change front (Kim et al. 2001b). The time interval is set to 0.01sec.

The evolution of streamlines is depicted in Fig. 3. At 18sec, the melt thickness reaches the critical thickness estimated from $Gr_{crit}=8000$ but the multi-cellular pattern cannot be identified. After the melting progresses further, at about 30sec, multiple flow cells in the melt become obvious. The melt region at 30sec corresponds to the cavity with the aspect ratio of 11.5. The delay of the onset of multi-cellular pattern can be explained by considering that the melt has been confined within a thin region until it grows to the critical thickness. The insufficient melt thickness can affect the process of the formation of multiple cells and it is expected that it should result in the delay of the onset of multi-cellular pattern. The number of cells varies with time, beginning with five cells at 30sec, and increasing to six at 42sec, then decreasing five again at 48sec, four and finally to one major cell accompanied by several minor cells. Compared with the results of Danzig (1989), Stellar and Giangi (2000), and Wintruff et al. (2001) who conducted numerical investigations on the present problem, the evolution pattern of multiple flow cells obtained in this work is somewhat different from theirs. While Danzig, Stellar and Giangi, and the present work predicted six flow cells around 30~40 sec after the onset of melting, Wintruff et al. observed four cells which were not identified in Danzig. Nevertheless, a consistent result by all of the works mentioned above except Danzig is that four-cell pattern, which emerges around 50~60 sec in the results of Stellar and Giangi and the present work, continues until 90~100 sec and succesively evolves into three-, two- and single-cell patterns. It appears that the discrepancy is still unresolved, but it may be related to the difference in numerical models to describe the phase change and the convection. It should be noted that the flow in the



(a) 18sec (b) 30sec (c) 36sec (d) 42sec (e) 48sec (f) 60sec

(g) 90sec (h) 2min (i) 4min (j) 8min

Fig. 3 Streamlines in the gallium melt. The dotted lines denote the reverse flows

melt can not preserve the symmetry due to the melting, while the flow in a rectangular cavity is essentially symmetrical about the center of the cavity provided the fluid properties are constant.

4. Conclusions

The convection-dominated melting in a rectangular cavity is investigated numerically. In particular, the multi-cellular flow structure in the melt region like a tall cavity that allows multiple flow cells is simulated and analyzed.

The melting including the formation and evolution of multi-cellular flow pattern in the melt is successfully simulated with the enthalpy-porosity method for the phase change process and with the central difference interpolation scheme for the convection term. The upwind difference scheme is not adequate to predict the detailed flow structure due to its over-diffusive characteristics. However, even rather coarse mesh points often give reasonable results for macroscopic behaviors like the location of phase interface and the volume fraction of the melt region. To observe the multi-cellular structure in the melt, high-resolution calculation using fine mesh points is needed. For instance, according to results of this study, more than 25 meshes in the width of the melt region are required to predict the multi-cellular structure at the moment of its inception. In the longitudinal direction, more than 15 mesh points per flow cell is usually needed.

The formation and evolution of the multiple flow cells in the melt region is approximately similar to that of a single-phase flow in a tall cavity with the same aspect ratio. However, the continuous change of the melt region due to the melting affects the detailed process. The critical thickness of the melt initiating the multi-cellular pattern is somewhat larger than that of pure natural convection in a cavity because the melt have remained in a narrower region until the thickness reaches the critical value. The non-uniform melt geometry, which features a wider melt zone in the top than in the bottom, gives rise to non-symmetrical melt-convection that is different from the natural convection observed in a cavity with fixed flow domain. Also, the comparison of the presented results and the previous ones shows that some after the onset of multiple cells the predicted evolution patterns agree among themselves and one of major controversial points is the initial evolution of the multi-cellular pattern immediately after the onset. Further investigations, numerical algorithms and especially experiments very carefully controlled to preserve multi-cells, are required to clarify the present problem.

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